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TOWARDS A CLIMATOLOGY OF SUPERCOOLED CLOUD WATER CONTENT

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1. INTRODUCTION

The Meteorological Office programme of research designed to provide data on the frequency of occurrence of conditions likely to represent an icing hazard to helicopters was outlined at the MOD(PE) Helicopter Icing Symposium on March 18, 1981.

A prime requirement is some estimate of the probability of encounter with various levels of supercooled cloud content over the NATO area up to 10,000 ft. It was indicated that progress in this regard was being made by combining cloud occurrence observations obtained on routine aircraft meteorological reconnaissance (RECCE) flights accumulated during and after World War II with observations of liquid water content currently being made by the Hercules aircraft of the Meteorological Research Flight based at RAE Farnborough.

This study will supersede the existing Meteorological Office publication IDM 105 "The Probability of Icing between 0 and 5000 ft" which, although of some value, is deficient in the following respects.

(i) IDM 105 simply indicated the probability of being in $\frac{5}{8}$ or more supercooled cloud with no indication of cloud water content

(ii) IDM 105 was based on surface observations of cloud (cloud amount, type and height of base). Cloud thickness had to be estimated, and the values adopted appear to be overestimates.

(iii) IDM 105 covers height variation in two coarse bands, 0-1000 ft and 2000-5000 ft, with no information above 5000 ft.

The current work should remedy all these deficiencies, and the purpose of this work is to describe the method being used to obtain a climatology of supercooled water content in more detail than given at the March 18 Symposium.

2. EVALUATION OF PROBABILITIES

At a given height, location and season, the probability, P(sq), of encountering

supercooled liquid water contents exceeding a given level q is the product of the probability, P(sc), of the occurrence of supercooled cloud and the conditional probability, P(q/sc), of liquid water content exceeding q, given that cloud is present

$$P(s_q) = P(sc) \cdot P(q/sc) \qquad (1)$$

a. Evaluation of P(sc)

For climatological purposes, P(sc) may be derived directly from the RECCE data by summing the cloud amounts observed at a given level over an area large enough to include several observations in a given season for all occasions when temperature was $\leqslant 0^{\circ}C$.

P(sc) can be subdivided into its components, P(c), the probability that cloud is present, and P(s/c), the conditional probability that the cloud will be below $0^{\circ}C$. There is a need to know P(c) in a wider context than helicopter icing, and a search for possible correlations of P(c) with temperature anomaly and surface pressure at different levels in the atmosphere which might form the basis of cloud forecasting techniques is being made. This subdivision, however, is not really necessary for the climatology discussed here.

b. Evaluation of P(q/sc)

The most direct method of evaluating P(q/sc) would be to construct a cumulative frequency distribution of all obtainable liquid water content measurements, regardless of location in cloud, and read P(q/sc) from the resulting curve.

thus $$P(q/sc) = \int_{q}^{\infty} p(q')\,dq' \qquad q' \geq q \qquad (2)$$

where $p(q')$ is probability that l.w.c. lies between q' and $q' + dq'$

This however, would take no account of the observed relationship of q to height above cloud base and to its datum, the adiabatic water content q_{ad}, which on theoretical and observational grounds appears to form an upper limit to q. Thus the use of P(q/sc) which did not take account of this would indicate finite probabilities of large q near cloud base, for which there is no evidence in real clouds.

Thus P(q/sc) has to be related to height h, above cloud base and is defined by

$$P(q/sc) = \int_0^z p(h) P_h(q/sc)\,dh \qquad z \geq h \geq 0 \tag{3}$$

where p(h) is probability that level z lies between h and h + dh above cloud base, and

$$P_h(q/sc) = \int_q^{q_{ad}(h)} p(q')\,dq' \tag{4a}$$

where the upper limit of the integral is now q_{ad}, itself a function of height above cloud base. It follows that

$$P_h(q/sc) = 0 \quad \text{for} \quad q \geq q_{ad}(h)$$

and precludes the possibility of large liquid water content near cloud base.

A more convenient representation may be to transform the variable q' on the righthand side of Eq 4a to $\alpha = q'/q_{ad}$; $\alpha_c = q/q_{ad}$ to give

$$P_h(q/sc) = \int_{\alpha_c(h)}^{1} p(\alpha)\,d\alpha \qquad 1 \geq \alpha \geq \alpha_c \tag{4b}$$

There is some evidence that the form of $p(\alpha)$ may be fairly insensitive to h in a given cloud type for at least the first km above cloud base. For values of h > 1 km, indications are that the form of $p(\alpha)$ becomes much more variable, and it may prove simpler to revert to the use of Eq. 2 in its basic form with no loss of accuracy at levels more than 1-2 Km above cloud base.

Some account will have to be taken of cloud type in the use of appropriate p(q') or $p(\alpha)$ functions. The basic division is between stratiform and cumuliform cloud type. Limited data on stratiform cloud has been produced by Met O 15 and is shown in Fig. 1. Near cloud base, a straight line intersecting the 50% level at $\alpha \sim 0.63$ is obtained. Well above cloud base, a curve results at generally lower values of α, including a few percent at $\alpha = 0$ (i.e. gaps in cloud). Recent observation in cumuliform cloud are not to hand, but early work by Squires and Warner suggest a similar decrease in mean α from about 0.6 near cloud base to 0.2 or less 1-2 Km above cloud base. It is likely, however,

that the more intense mixing in convective clouds will lead to lower values of α than in stratiform clouds at a given height above cloud base. This is reflected in schematic curves for cumulus in Fig. 1.

c. Computation of q_{ad}.

This is required to define the lower limit of the integral in Eq. 4b.

$$q_{ad}(h) = \rho(z)[q_s(z-h) - q_s(z)] \quad (5)$$

where $\rho(z)$ = air density at height z

$q_s(z)$ = saturation mixing ratio at height z

All the terms on the RHS are pressure and temperature dependent, so that in principle, Eq. 5 should be evaluated for every encounter with supercooled cloud.

However, the gradients $\partial q_{ad}/\partial p$ and $\partial q_{ad}/\partial h$ do not vary a great deal over the range of heights and temperatures likely to occur over the NATO area in winter. For instance, supercooled cloud cannot occur below 700 mb for wet bulb potential temperatures (θ_w) greater than about 15°C, and during the winter over the NATO area will not often lie outside the range 0-10°C.
It is proposed that we use

$$\rho(z) = c_1 p^*(z) = c_1 z^* \quad (6)$$
$$q_s(z-h) - q_s(z) = c_2[p^*(z-h) - p^*(z)] = c_2 h^*$$

where C_1 and C_2 are taken as constant coefficients representative of a value of θ_w of 5°C using the values shown in Table 1. $p^*(z)$ is pressure at height z.

TABLE 1: VARIATION OF $\begin{bmatrix} q_s\ (g\ kg^{-1}) \\ T\ (^\circ C) \\ \rho\ (kg\ m^{-3}) \end{bmatrix}^+$ with θ_w and p^*

θ_w (°C)	0	5	10
p^*(mb)			
	1.11	1.91	3.25
700	-20.0	-13.6	-6.9
	0.963	0.940	0.916
	1.87	2.98	4.75
800	-12.2	-6.3	-0.1
	1.068	1.044	1.021
	2.79	4.22	6.17
900	-5.6	-0.1	5.2
	1.172	1.148	1.126
	3.82	5.47	7.73
1000	0.0	5.0	10.0
	1.275	1.252	1.230

+ values obtained from appropriate saturated adiabat on tephigram

This gives

$$q_{cld} = 1.61 \times 10^{-5}\ z^*\ h^* \qquad (7)$$

or $$\alpha_c = \frac{6.21 \times 10^{-4} q}{z^* h^*}$$

where z^* and h^* are in mb (Eq. 6)

d. Computation of P(q/sc)

It is suggested that the ensemble of THUM observations can be used to evaluate characteristic curves of $p(h)$ at pressure levels of 700, 800 and 900 mb with $p(h)$ tabulated at 10 mb intervals. It may be necessary to distinguish between stratiform and cumuliform cloud and between land and sea profiles. It is suggested that levels of q be chosen at 0.2, 0.5 and 1 gm^{-3} initially.

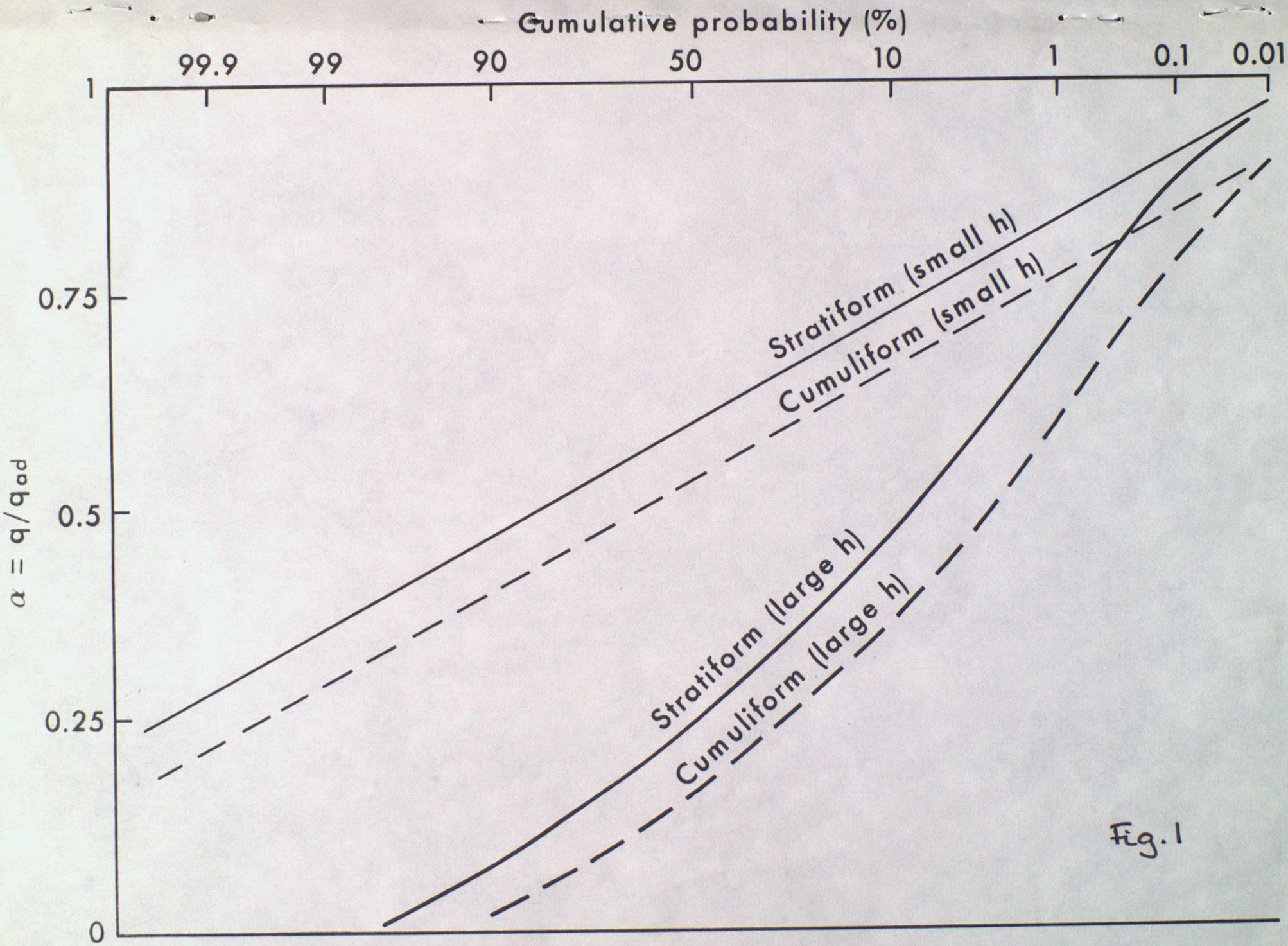
Cumulative probability (%)
99.9
99
90
50
10
1
0.1
0.01
1
0.75
0.5
0.25
0
$\alpha = q/q_{ad}$
Stratiform (small h)
Cumuliform (small h)
Stratiform (large h)
Cumuliform (large h)

Fig. 1